Name: _____

Teacher: _____

Class: _____

2014
HIGHER SCHOOL CERTIFICATE COURSE
ASSESSMENT TASK 3: TRIAL HSC

Mathematics

Time allowed: 3 hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
	Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
H2, H3, H4, H5	Manipulates algebraic expressions to solve problems from topic areas such as geometry, co-ordinate geometry, quadratics, trigonometry, probability and logarithms	11, 12
H6, H7, H8	Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	13, 14
H9	Synthesises mathematical solutions to harder problems and communicates them in appropriate form	15, 16

Total Marks 100

Section I   10 marks
Multiple Choice, attempt all questions,
Allow about 15 minutes for this section

Section II   90 Marks
Attempt Questions 11-16,
Allow about 2 hours 45 minutes for this section

General Instructions:

- Questions 11-16 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 90	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
Q15	/15	
Q16	/15	
	Percent	

Section 1   **Multiple Choice (10 Marks)**   **Circle Correct Answer**

Question 1

Find $\log_4 32$

a) 1.5

b) 2

c) 2.5

d) 3

Question 2

Solve $x^2 + 4x - 1 = 0$

a) $x = -2 \pm \sqrt{5}$

b) $x = 2 \pm \sqrt{5}$

c) $x = -2 \pm 2\sqrt{5}$

d) $x = -4 \pm \sqrt{5}$

Question 3

Find the range of $y = 3 + 2\cos(2x - 3)$

a) $-2 \le y \le 2$

b) $-\frac{3}{2} \le y \le \frac{2}{3}$

c) $3 \le y \le 5$

d) $1 \le y \le 5$

Question 4

What is the derivative of $\dfrac{2x}{1+x^2}$

a) $\dfrac{2-x^2}{(1+x^2)^2}$

b) $\dfrac{2+2x^2}{(1+x^2)^2}$

c) $\dfrac{2-2x^2}{(1+x^2)^2}$

d) $\dfrac{-2-2x^2}{(1+x^2)^2}$

Question 5

What are the solutions of

$2\cos\theta = -\sqrt{3} \; for \; 0 \le \theta \le 2\pi$?

a) $\dfrac{\pi}{6}$ and $\dfrac{5\pi}{6}$

b) $\dfrac{5\pi}{6}$ and $\dfrac{7\pi}{6}$

c) $\dfrac{7\pi}{6}$ and $\dfrac{11\pi}{6}$

d) $\dfrac{\pi}{6}$ and $\dfrac{7\pi}{6}$

Question 6

What is the value of $\int_{5}^{15} \frac{1}{5x} \, dx$

a) $\frac{1}{5} ln5$

b) $\frac{1}{5} ln3$

c) $\frac{1}{5} ln10$

d) $\frac{3}{5} ln5$

Question 7

What is the perpendicular distance of the point (3, -2) from the line $y = 4 - 3x$

a) $\frac{4}{\sqrt{10}}$

b) $\frac{15}{\sqrt{10}}$

c) $\frac{3}{\sqrt{10}}$

d) $\frac{7}{\sqrt{10}}$

Question 8

The solution to $(2x - 5)(6 - x) \geq 0$ is

a) $\{x : -2.5 \leq x \leq 6\}$

b) $\{x : 2.5 \leq x \leq 6\}$

c) $\{x : x \leq 2.5 , x \geq -6\}$

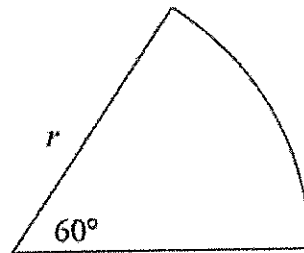d) $\{x : x \leq -2.5 , x \geq 6\}$

Question 9

For what values of x is the curve $y = 4x^3 - 3x^2$ concave down?

a) $x > \frac{1}{4}$

b) $x < \frac{1}{4}$

c) $x > \frac{3}{4}$

d) $x < 0$

Question 10

The sector below has an area of 30π square units

Not to scale

The value of r is

a) $5\sqrt{6}$

b) $\frac{6}{\sqrt{5}}$

c) $6\sqrt{5}$

d) $3\sqrt{2}$

Section II

90 Marks

Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.
In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations

Question 11 (15 Marks) Use a SEPARATE writing booklet

a) Factorise $3x^2 - 16x + 5$.   **2**

b) Solve $|5x + 2| < 3$ **2**

c) Differentiate $(3 - cos2x)^5$ **2**

d) Find the coordinates of the focus of the parabola   **2**
$x^2 = 20(y + 3)$

e) Find the equation of the normal to the curve $y = \dfrac{2}{x}$ **3**
At the point where x=3.

f) Evaluate $\int_1^3 \dfrac{4}{x^3} \, dx$ **2**

g) Sketch the region $(x + 3)^2 + (y - 2)^2 \geq 16$ **3**
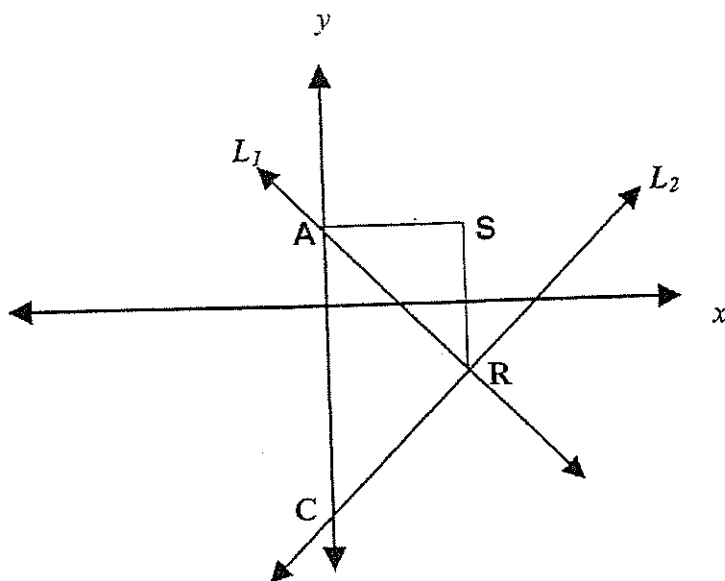
2

Question 12 (15 Marks) Use a SEPARATE writing booklet

a) Find the equation of the tangent to $y = x\cos x$ where $x = \frac{\pi}{2}$ 3

b)

Line L_1 has equation $x + y = 4$ and intersects the y axis at point A.
Line L_2 has equation $x - y = 8$ and intersects the y axis at point C.
L_1 and L_2 Intersect at point R.

The horizontal line through A intersects the vertical line through R, at S.

(i)   Find the coordinates of point A and C.   2
(ii)   Show that R has coordinates (6, -2).   1
(iii)   State the equation of the line SR.   1
(iv)   Find the gradient of the line L_1.   1
(v)   Find the distance AR.   1
(vi)   Show that triangle ARC is a right-angled isosceles triangle.   2
(vii)   Find the equation of the circle with centre R, passing through
        the points A and C.   2

c) Sketch the graph of $y = 4\cos x$ for $0 \le \theta \le 2\pi$ 2

Question 13 (15 Marks) Use a SEPARATE writing booklet
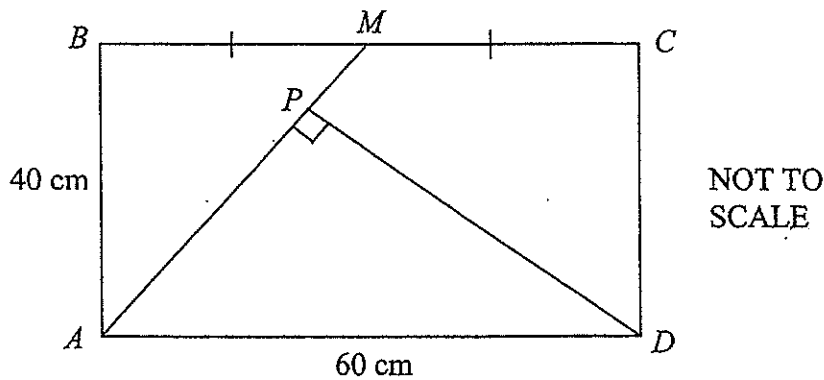
a) Differentiate with respect to x:

    (i) $x\sqrt{x}$ 1

    (ii) $x^2 \ln x^2$ 2

    (iii) $\dfrac{e^{-2x}}{\sin 3x}$ 2

b) Find $\displaystyle\int \dfrac{3\sec^2 2x}{1+\tan 2x}\, dx$ 2

c)

$ABCD$ is a rectangle in which $AB = 40$ cm and $AD = 60$ cm. M is the midpoint of BC and DP is perpendicular to AM.

Draw a neat sketch on your answer sheet. Hence:

    (i)   Prove that triangles ABM and APD are similar.   2

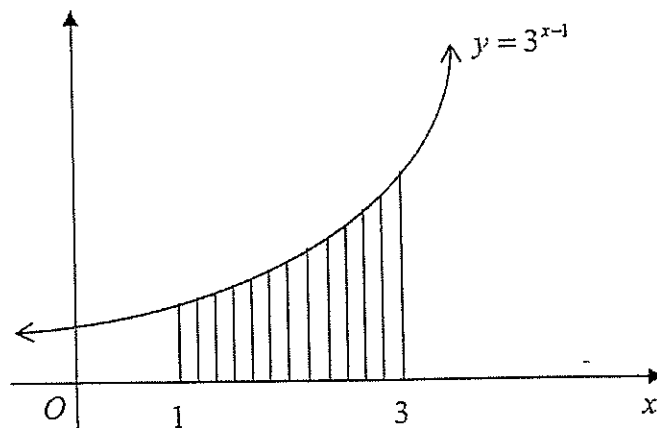   (ii)   Calculate the length of PD.   2

    (iii)   Using Pythagoras' Theorem in triangle APD show that $AP = 36$ cm.   1

    (iv)   By finding the two areas of the triangles ABM and APD, prove that the area of the quadrilateral $PMCD$ is 936 cm^2.   3

Question 14 (15 Marks) Use a SEPARATE writing booklet

a)

The diagram below shows the shading of a region bounded by the graph $y = 3^{x-1}$ and the lines $x = 1$ and $x = 3$.

(i)   Copy and complete the following table giving your answer correct to three decimal places: **1**

x	1	1.5	2	2.5	3
$y = 3^{x-1}$	1	1.732			

(ii)   Use Simpson's Rule with five function values to approximate the shaded area to three decimal places.   **2**

b) Consider the curve given by the equation $y = x^3 - 6x^2 + 9x + 4$

(i)   Find the coordinates of the stationary points and determine their nature   **4**

(ii)   Find the coordinates of any point of inflexion.   **2**

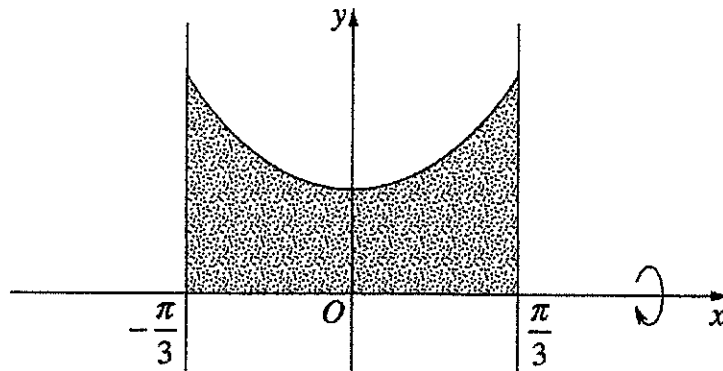(iii)   Sketch the curve, showing only the above information.   **2**

(iv)   Determine the values of x for which $\frac{dy}{dx} > 0$ **1**

c)

The diagram shows the region bounded by the curve $y = \sec x$, the lines $x = \dfrac{\pi}{3}$ and $x = -\dfrac{\pi}{3}$, and the x-axis.

3

The region is rotated about the x-axis. Find the volume of the solid of revolution formed.

End of Question 14

Question 15 (15 Marks) Use a SEPARATE writing booklet

a)

On being retrenched from his job, Kevin receives a cash payment of $20 000.

One year later, he receives his first annual payout of $10 000. He continues to receive annual payouts of $10 000 every year thereafter.

He places all of this money in his suitcase as he receives it, and spends none.

At the end of every year, just before the next payout, Kevin spends 20% of the money in his suitcase on a holiday.

Let A_n be the amount Kevin has in his suitcase immediately after his n^{th} annual payout.

(i) Show that Kevin has $26 000 in his suitcase immediately after his first annual payout. 1

(ii) Show that the money in Kevin's suitcase immediately after his 3rd annual payout is given by 2

$$A_3 = 20000(0.8)^3 + 10000\left(1+0.8+0.8^2\right).$$

(iii) Show that $A_n = 50000 - 30000\left(0.8^n\right)$. 3

(iv) After how many years will the amount in Kevin's suitcase first exceed $48 000? 2

(v) What is the most money Kevin will ever have in his suitcase? 1

b)

Two particles, A and B, move along a straight line so that their displacements, x_A and x_B, in metres, from the origin at time t seconds are given by the following equations respectively:

$$x_A = 12t + 5 \qquad\qquad x_B = 6t^2 - t^3$$

(i) Find two expressions for the velocities of particles A and B. 2

(ii) Which of the two particles is travelling faster at $t = 1$ second? 1

(iii) At what time does particle B come to rest? 1

(iv) Find the maximum positive displacement of particle B. 2

Question 16 (15 Marks) Use a SEPARATE writing booklet

a) A 250mg tablet is dissolved in a glass of water. After t minutes the amount of undissolved tablet, U in mg, is given by the formula:

$U = 250 \, e^{-kt}$, where k is a constant.

(i) Calculate the value of k, correct to 4 decimal places, given that 10mg of the tablet remain after 15 minutes. **2**

(ii) Find the rate at which the tablet is dissolving in the glass of water after 10 minutes.
Give your answer correct to two decimal places. **2**

b)
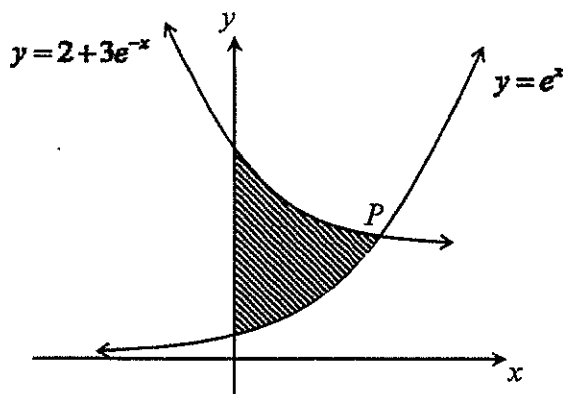
The diagram shows the graphs of $y = e^x$ and $y = 2 + 3e^{-x}$ intersecting at the point P.

(i) Show that the curves intersect when **1**

$$e^{2x} - 2e^x - 3 = 0 .$$

(ii) Hence show that the x-coordinate of the point P is $\ln 3$. **2**

(iii) Hence find the exact area of the shaded region. **3**

Question 16 continues on the next page

Question 16 continued

c) There are 5 red marbles and 4 blue marbles in a bag. Bill and Ben are playing a game in which they take turns drawing a marble from the bag and then replacing it.

To win the game, Ben must draw a red marble and for Bill to win he must draw a blue marble. They continue taking turns until there is a winner. Ben goes first.

(i)   Find the probability that Ben wins on his first draw.   **1**

(ii)   Find the probability that Ben wins in three or less of his turns.   **2**

(iii)   Find the probability that Ben wins the game.   **2**

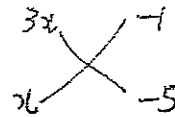**End of Question 16**

END OF EXAMINATION

Q1	C		Q6	B
Q2	A		Q7	C
Q3	D		Q8	B
Q4	C		Q9	B
Q5	B		Q10	C

Section II

Q11

a) $3x^2 - 16x + 5$ $\therefore (3x-1)(x-5)$ [2]

Well done

b) $|5x+2| < 3$ $\therefore \pm(5x+2) < 3$

$\oplus$ $5x + 2 < 3$ $\ominus$ $-5x - 2 < 3$

$5x < 1$ $-5x < 5$

$x < \frac{1}{5}$ $x > -1$

Soln $\left\{ x : -1 < x < \frac{1}{5} \right\}$

Some students failed to swap inequality, due to negative sign, in 2nd inequality [2]
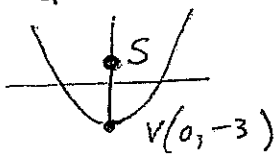
c) $\frac{d}{dx}(3 - \cos 2x)^5 = 5(3 - \cos 2x)^4 \times -(-2\sin 2x)$

$= 10 \sin 2x (3 - \cos 2x)^4$

Some students forgot to multiply by -2 [2]

d) Focus of $x^2 = 20(y+3)$ $\therefore 4a = 20$ i.e $a = 5$

$\therefore$ $S(0, 2)$ (focus)

Well done   [2]

$V(0, -3)$

e) Eqⁿ Normal $y = \frac{2}{x}$ at $x = 3$ $\therefore y = \frac{2}{3}$

$y' = -2x^{-2} = \frac{-2}{x^2}$ at $x = 3$ $\therefore y'(3) = \frac{-2}{9}$

$\therefore$ Normals gradient $\frac{9}{2}$ $\therefore$ eqⁿ Normal $y - \frac{2}{3} = \frac{9}{2}(x - 3)$

$\therefore y = \frac{9}{2}x - \frac{77}{6}$ or $27x - 6y - 77 = 0$

Many students thought $y' = 2\ln x$.   [3]

f) $\int_1^3 \frac{4}{x^3} dx = \int_1^3 4x^{-3} dx = \left[\frac{4x^{-2}}{-2}\right]_1^3 = \left[\frac{-2}{x^2}\right]_1^3$

$= -\frac{2}{9} + 2 = \boxed{1\frac{7}{9}}$

Some students differentiated $\frac{4}{x^3}$ instead of integrating.   [2]

g) Centre $(-3, 2)$
radius $r = 4$
TP$(-3, 0)$ False

1 for circle correct centre + radius
1 (solid circle region
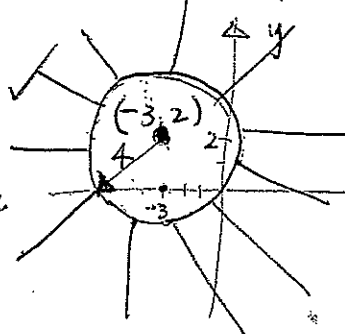
Some students did not shade correctly.   [2]

Q12

a) $y = x \cos x$ at $x = \frac{\pi}{2}, \ y = 0$

$y' = x \cdot -\sin x + \cos x \cdot 1 = -x\sin x + \cos x$ ✓

$y'\left(\frac{\pi}{2}\right) = -\frac{\pi}{2} \cdot 1 + 0 = -\frac{\pi}{2}$ ✓   [3]

∴ Eqⁿ of Tangent $y - 0 = -\frac{\pi}{2}\left(x - \frac{\pi}{2}\right)$

∴ $y = -\frac{\pi}{2}x + \frac{\pi^2}{4}$ or $2\pi x + 4y - \pi^2 = 0$ ✓

b) $L_1 : x + y = 4$ ∴ $L_1 + L_2 \rightarrow 2x = 12$ ∴ $x = 6$

$L_2 : x - y = 8$ Sub $x = 6$ into L_1 ∴ $y = -2$

(i) $L_1 : x = 0 \ y = 4$ (ii) See above (Solve simultaneously)

    $L_2 : x = 0 \ y = -8$ $R(6, -2)$ ✓

∴ $A(0, 4)$ ✓   (iii) SR $\boxed{x = 6}$ ✓

   $C(0, -8)$ ✓

(iv) $L_1 \rightarrow y = -x + 4$ (v) AR $(0, 4)$ to $(6, -2)$

   ie $\boxed{M = -1}$ ✓ $d = \sqrt{(6-0)^2 + (-2-4)^2} = \sqrt{36 + 36}$

                     $d = \sqrt{72}$ or $6\sqrt{2}$ ✓   [10]

(vi) $AR = 6\sqrt{2}$ $\overline{RC}, (6, -2)$ to $(0, -8)$

                $d = \sqrt{(6-0)^2 + (-2+8)^2} = \sqrt{36 + 36}$

                $d = \sqrt{72}$ or $6\sqrt{2}$ ✓

∴ $\triangle ARC$ is isosceles as $\overline{RC} = \overline{AR} = 6\sqrt{2}$

$m_{L_1} = -1, \ m_{L_2} = 1$ as $m_{L_1} \times m_{L_2} = -1$ ∠R is $90°$ ✓

∴ $\triangle ARC$ is right-angled isosceles triangle. ✓

(vii) $r = 6\sqrt{2} = \sqrt{72}$ ∴ eqⁿ of circle centre $(6, -2) \ r = \sqrt{72}$

   is $(x-6)^2 + (y+2)^2 = 72$ ✓

c) $y = 4\cos x$

   [2]
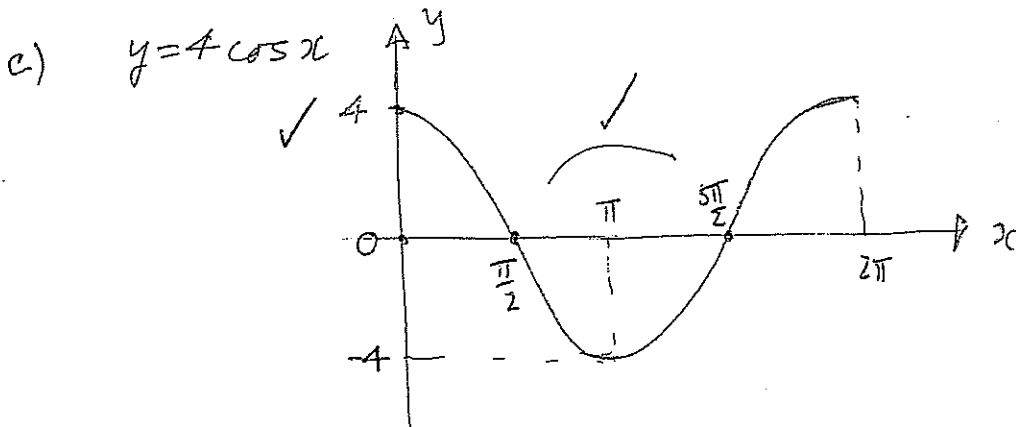
a) (i) $y = x^{\frac{3}{2}}$; $y' = \frac{3}{2} x^{\frac{1}{2}}$ ✓ ☐1

(ii) $y = x^2 \ln x^2$; $y' = x^2 \cdot \frac{2}{x} + 2\ln x \cdot 2x$ ☐2
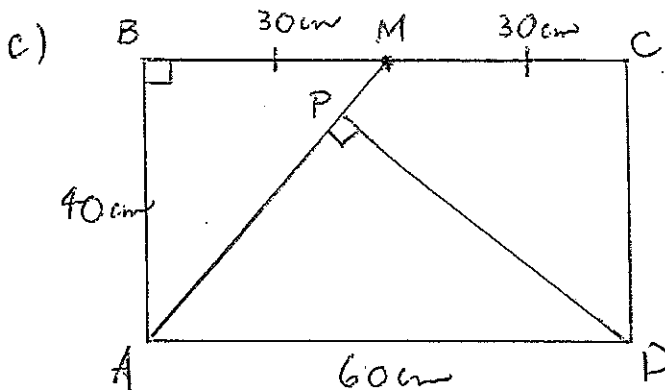product rule

∴ $y' = 2x + 4x \ln x$ or $2x(1 + 2\ln x)$.

(iii) $y = \dfrac{e^{-2x}}{\sin 3x}$ $y' = \dfrac{\sin 3x \cdot -2e^{-2x} - e^{-2x} \cdot 3\cos 3x}{\sin^2 3x}$ ✓
Quotient rule.

$y' = \dfrac{-e^{-2x}(2\sin 3x + 3\cos 3x)}{\sin^2 3x}$ ✓ ☐2

b) $\displaystyle\int \frac{3\sec^2 2x}{1 + \tan 2x} dx = \frac{3}{2} \int \frac{2\sec^2 2x}{1 + \tan 2x} dx$ ✓

$= \frac{3}{2} \ln(1 + \tan 2x) + C$ ✓ ☐2

c)

(i) Prove $\triangle ABM \,|||\, \triangle APD$

① $\angle B = \angle P$ (90° given)

② $\angle BAM + \angle AMB = 90°$
(angle sum of $\triangle$)

also $\angle BAM + \angle PAD = 90°$ (rectangle

∴ $\angle AMB = \angle PAD$

(or alternate) → 3rd angle equal

∴ $\triangle ABM \,|||\, \triangle APD$ (equiangular) ☐2

(ii) PD ? → AM = 50 cm (Pythagoras)

∴ $\dfrac{50}{60} = \dfrac{40}{PD}$ (corresponding sides similar $\triangle$s)

∴ $PD = \dfrac{40 \times 60}{50} = 48$ cm ☐2

(iii) $AP^2 = AD^2 - PD^2 = 60^2 - 48^2$

∴ $AP^2 = 1296$ ∴ $AP = \sqrt{1296} = 36$ cm ✓ ☐1

(iv) Area of PMCD $= \overset{Total}{40 \times 60} - [\overset{sum \, of \, 2 \, \triangle s}{\frac{1}{2} \times 40 \times 30 + \frac{1}{2} \times 36 \times 48}]$ ✓

$= 2400 - [600 + 864]$ ✓

$= 936$ cm^2 ✓ ☐3

Q14

a) (i)
$$y = 3^{x-1} \quad (3DP) \quad \boxed{1}$$

x	1	1.5	2	2.5	3
y	1	1.732	3	5.196	9

Well done

(ii)
$$h = 0.5 \text{ (width)}$$

$$\int_1^3 3^{x-1}\, dx \doteq \frac{h}{3}\left[1^{st} + last + 4\,odds + 2\,evens\right]$$

$$\doteq \frac{0.5}{3}\left[1 + 9 + 4[1.732 + 5.196] + 2 \times 3\right]$$

$$\doteq \frac{1}{6}\left[10 + 6 + 27.712\right] \doteq \frac{1}{6} \times 43.712$$

$$= \underline{7.285} \quad (3DP) \quad \boxed{2}$$

Some wrong formulas
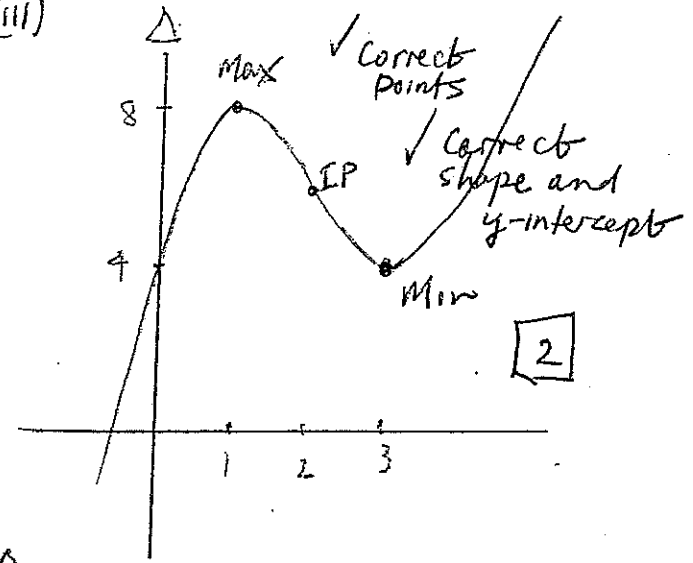
b) $y = x^3 - 6x^2 + 9x + 4$

(i)
$$y' = 3(x^2 - 4x + 3)$$
$$= 3(x - 3)(x - 1)$$

$$f'(x) = 0 \text{ when } x = 1, 3$$

$$y'' = 6x - 12$$

when $x = 1$, $y = 8$, $f'(1) = 0$
and $f''(1) = -6 < 0$

$\therefore$ $(1, 8)$ is a max turning point

when $x = 3$, $y = 4$, $f'(3) = 0$
and $f''(3) = 6 > 0$

$\therefore$ $(3, 4)$ is a min turning pt

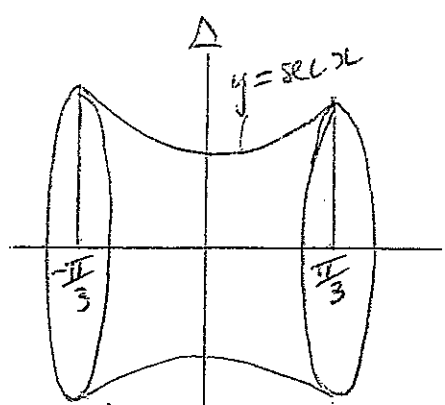(ii) <u>Inflexions</u> $f''(x) = 0$ when $6x - 12 = 0$
$\therefore$ $f''(2) = 0$ check for concavity change

$$f''(2-\epsilon) = f''(1) < 0 \;]$$
$$f''(2+\epsilon) = f''(3) > 0 \;] \quad \therefore (2, 6) \text{ inflexion}$$

Some students did not test for P.O.I

(iii)

✓ Correct Points
✓ Correct shape and y-intercept

$\boxed{2}$

Max, IP, Min labeled; axes marked 8, 4 on y-axis and 1, 2, 3 on x-axis

(iv) $\frac{dy}{dx} > 0$ when
$$x > 3, \; x < 1 \quad \boxed{1}$$
mostly well done

c)

$y = \sec x$, from $-\frac{\pi}{3}$ to $\frac{\pi}{3}$

As $y = \sec x$ is even ie $\sec(-\frac{\pi}{3}) = \sec(\frac{\pi}{3})$

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \to 2\int_0^{\frac{\pi}{3}}$$

$$V = \pi \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} [f(x)]^2\, dx \;\checkmark$$

$$\therefore V = 2\pi \int_0^{\frac{\pi}{3}} \sec^2 x\, dx$$

$$= 2\pi\left[\tan x\right]_0^{\frac{\pi}{3}} \checkmark$$

$$= 2\pi\left[\tan \frac{\pi}{3} - \tan 0\right]$$

$$V = 2\sqrt{3}\,\pi \; u^3 \quad \checkmark \quad \boxed{3}$$

Well done

Use exact values where possible

(triangle with sides $\sqrt{3}$, 1, and angle $\frac{\pi}{3}$, base 1)
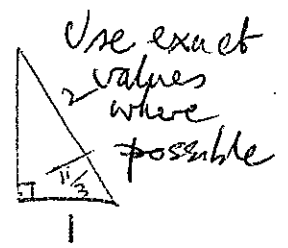
Q15 a) (i) $A_1 = 20000 \times 0.8 + 10000 = \$26,000$ ☐1

(ii) $A_2 = A_1 \times 0.8 + 10,000$

$\qquad = [20000 \times 0.8 + 10000] \times 0.8 + 10,000$

$\qquad = 20000(0.8)^2 + 10000[1 + 0.8]$ ✓ Well done

$A_3 = A_2 \times 0.8 + 10000$

$\qquad = [20000(0.8)^2 + 10000[1+0.8]] \times 0.8 + 10000$

$\qquad = 20000(0.8)^3 + 10000[0.8 + 0.8^2] + 10000$ ☐2

$\qquad = 20,000(0.8)^3 + 10000[1 + 0.8 + 0.8^2]$ as required

(iii) $A_n = 20,000(0.8)^n + 10000[1 + 0.8 + 0.8^2 + \cdots + 0.8^{n-1}]$ ✓

$\qquad = 20000(0.8)^n + 10000 \left[S_n = \dfrac{1(1-(0.8)^n)}{1-0.8} \right] \to S_n = \dfrac{a(1-r^n)}{1-r}$

$\qquad = 20000(0.8)^n + 50000(1-(0.8)^n)$ Mostly well done

$\qquad = 50000 - 30000(0.8)^n$ ✓   ☐3

(iv) $50000 - 30000(0.8)^n > 48000$

$\qquad -30000(0.8)^n > -2000$

$\qquad \therefore (0.8)^n < \frac{2}{30}$ take logs $n \log 0.8 < \log \frac{1}{15}$ ✓

and as $\log(0.8) < 0$ $\quad \therefore \quad n > \dfrac{\log\left(\frac{1}{15}\right)}{\log(0.8)} > 12.1359\ldots$

Some inequality signs wrong. $\quad \therefore \boxed{n = 13 \text{ yrs}}$ $\left(\begin{array}{l}\text{required} \\ \text{to exceed } \$48,000\end{array}\right)$ ✓   ☐2

(v) As $n \to \infty$ $\quad A_n = 50000 - 30000(0.8)^n$

$\qquad 0.8^n \to 0$ $\quad \therefore A_n \to \underline{50,000}$ $\left(\begin{array}{l}\text{most kevin will ever} \\ \text{have in suitcase}\end{array}\right)$ ☐1

✓

b) $x_A = 12t + 5$ ✓ $\qquad x_B = 6t^2 - t^3$ ✓

(i) $\therefore V_A = \dfrac{dx_A}{dt} = 12 \text{ m/s}$ $\quad \therefore V_B = \dfrac{dx_B}{dt} = 12t - 3t^2 \text{ m/s}$ ☐2

(ii) $V_A(1) = 12 \text{ m/s}$ $\quad V_B(1) = 12 - 3 = 9 \text{ m/s}$

$\qquad \therefore$ Particle A travelling faster at $t = 1$ sec ✓   ☐1

(iii) $V_B = 0$ $\quad \therefore 12t - 3t^2 = 0$ ie $3t(4-t) = 0$

$\qquad \therefore \underline{t = 0, 4 \text{ secs}}$ Particle B at rest $t = 0, 4$ secs ☐1

(iv) Maximum displacement $(\overset{>0}{\text{pos}})$ of particle B

$\qquad x_B(4) = 6 \cdot 4^2 - 4^3 = \underline{32 \text{ m}}$

$\qquad (V_B = 0)$ ✓   ✓   ☐2

Well done overall

Q16 a) $U = 250e^{-kt}$ (i) $10 = 250e^{-15K}$

$t = 15$ mins
$U = 10$ mg

$\ln e^{-15k} = \ln \frac{t}{25}$
$-15k = \ln 25$
$k = -\frac{1}{15}$

$\frac{dU}{dt} = -250ke^{-kt}$

errors
— wrong substitution
— incorrect calc. work

$\therefore \frac{1}{25} = e^{-15k}$ (take log$_e$)

$K = -\frac{1}{15}\ln\frac{1}{25} = -\frac{1}{15}\ln 5^{-2} = \frac{2}{15}\ln 5$

$\therefore k = 0.2146 \checkmark$ (4DP) $\checkmark$

$\boxed{2}$

(ii) $\frac{dU}{dt}$ $(t=10)$

$\frac{dy}{dt} = -ku$ decay

$\therefore \frac{dU}{dt} = -250 \cdot \frac{2}{15}\ln 5 \, e^{-10\times\frac{2}{15}\ln 5} = 3.90 \text{ mg/min}$

$\underline{-6.27}\checkmark$

$\boxed{2}$

b) When $y = e^x$ and $y = 2 + 3e^{-x}$ intersects

(i) $\boxed{e^x = 2 + 3}$
must be explicit

$\therefore e^{2x} = 2e^x + 3$ or $e^{2x} - 2e^x - 3 = 0$

$2 + 3e^{-x} = e^x \uparrow$

$\boxed{1}$

(ii) let $u = e^x$: $u^2 - 2u - 3 = 0$ ie $(u-3)(u+1) = 0$

$\therefore e^x = 3$ or $\boxed{e^x = -1}$ no soln

$\checkmark$

Take $\log_e$: $\ln e^x = \ln 3$ ie $x\ln e = \ln 3$ (as $\ln e = 1$)

$\therefore \underline{x = \ln 3}$ generally well answ. $\checkmark$

$\boxed{2}$

(iii) Exact Area $A = \displaystyle\int_0^{\ln 3} 2 + 3e^{-x} - e^x \, dx$ $\checkmark$

ie $A = \left[2x + 3e^{-x} - e^x\right]_0^{\ln 3} = (2\ln 3 - 3e^{-\ln 3} - e^{\ln 3}) - (0 - 1)$

$\therefore A = 2\ln 3 - 3\left[\frac{1}{3}\right] - 3 + 4$

many students could $\boxed{3}$
not correctly evaluate $-3e^{-\ln 2}$

$A = 2\ln 3$ $\checkmark$

c) $5R, 4B$

Ben Wins in 3 or less turns

$\substack{BL \\ Be}$ $\substack{BL \\ Bi}$ $\substack{BW}$ $\substack{BL \\ Be}$ $\substack{BL \\ Bi}$ $\substack{BL}$ $\substack{BL}$ $\substack{BW \\ Be}$

(i) $\frac{5}{9}$ $\checkmark$ (ii) $\frac{5}{9} + \frac{4}{9}\cdot\frac{5}{9}\cdot\frac{5}{9} + \frac{4}{9}\cdot\frac{5}{9}\cdot\frac{4}{9}\cdot\frac{5}{9}\cdot\frac{5}{9}$ $\checkmark$

$\boxed{1}$

$= \frac{5}{9} + \left(\frac{5}{9}\right)^2\cdot\frac{4}{9} + \left(\frac{5}{9}\right)^3\cdot\left(\frac{4}{9}\right)^2$ $\checkmark$ $\boxed{2}$

no $r = \frac{5}{9}\cdot\frac{4}{9} = \frac{20}{81}$

$= \frac{505}{729} + \frac{2000}{59049} \approx 0.72659994 \ldots 2$

Many student could not generate the seq.

(iii) Ben Wins $S_\infty = \dfrac{\frac{5}{9}}{1-\frac{20}{81}} = \frac{5}{9}\cdot\frac{81}{61} = \frac{45}{61}$ $\checkmark$

$\boxed{2}$

$P(\text{Ben Wins}) = \frac{45}{61} \approx 0.737704918$ $\checkmark$

many student
did not realise $\cancel{the}$ S_∞